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COSINE SIMILARITY APPROACHES TO RELIABILITY OF LIKERT SCALE AND ITEMS

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Abstract

Based on cosine similarities, the paper proposes two non-parametric methods of finding reliability of Likert items and Likert scale from single administration of the questionnaire, considering only the frequency or proportion for each cell of the Item-Response category matrix without involving any assumptions of continuous nature or linearity or normality for the observed variables or the underlying variable being measured. Each method enables to find reliability irrespective of distribution of the observed or underlying variables and avoiding test of uni-dimensionality or assumption of normality for Cronbach's alpha or bivariate normality for polychoric correlations. The proposed methods thus are considered as improvement over the existing ones. Reliability as per Bhattacharyya's measure appears to be preferred over the Angular Association method as the former expresses test reliability as a function of item reliabilities. In addition to offering the computational steps, empirical verification with real data is given to illustrate the concepts and usefulness of the proposed non-parametric reliability procedures.

Keywords: *Reliability; item-response category matrix; polychoric correlation; angular association; Bhattacharyya's measure.*

1. Introduction

Estimation of reliability of a Likert scale by most of the existing methods with different sets of assumptions deviates differently and thus gives different values for a single Likert scale. Reliability in terms of product moment correlation assumes at least

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interval measurement of the variables; continuous and normally distributed data. Cronbach's alpha makes additional, assumption of uncorrelated errors.

The assumptions are generally not satisfied by data generated from Likert scale. If the assumption of continuous nature of data and normality are violated, the variance -covariance matrix can be substantively distorted especially when two variables manifest themselves in skewed distribution of observed responses (e.g., Flora and Curran, 2004). Sheng and Sheng (2012) observed that skewed distributions produce a negative bias when the coefficient alpha is calculated. Green and Yang (2009) found similar results in an analysis of the effects of non-normal distributions in estimating reliability. Value of Cronbach's alpha can be increased by adding more number of items. However, increase in alpha on deletion of few items is common. Streiner (2003) observed that too high value of alpha probably indicates redundancy of items. Cronbach's alpha has been repeatedly misinterpreted and misused, Sijtsma, 2009). Limitations of this method have also been reported by Eisinga, Te Grotenhuis & Pelzer (2012) and Ritter (2010). The level of scaling obtained from Likert procedure is clearly at least ordinal. Response categories tend to be sequential but not linear. In order to achieve an interval scale, distance between a successive pair of response categories must be same. But it seems unlikely that the categories formed by the misalignment of a finite number of responses will all be equal. Thus, the interval scale assumption seems unlikely to hold. Parametric statistical methods like factor analysis, hierarchical linear models, structural equation models, t-test, ANOVA, etc. are based on assumption of normally distributed interval-level data. Similarly, generalizability theory based on ANOVA requires satisfaction of those assumptions. Lantz (2013) observed that respondents generally did not perceive a Likert-type scale as equidistant. A number of methods for "rescaling" ordinal scales to get interval properties have been proposed (e.g. Granberg-Rademacker, 2010; Wu, 2007; King et al., 2003). But use of such methods in practical analysis of Likert-type data seems to be rare. In addition to interval properties, assumptions regarding normality and homoscedasticity also need to be addressed. Chien-Ho Wo (2007) observed that transformation of Likert-scale data to numerical scores based on Snell's (1964) scaling procedure does not do much to pass the normality test. Granberg-Rademacker, (2010) proposed Monte Carlo Scaling method based on multivariate normal distribution. Muraki (1992) observed that if the data fits the Polytomous Rasch Model and fulfill the strict formal axioms of the said model, it may be considered as a basis for obtaining interval level estimates of the continuum.

Gadderman, Guhn and Zumbo (2012) proposed ordinal alpha for ordinal data based on the polychoric correlation matrix and defined ordinal alpha as $\alpha =$

$(\frac{p}{p-1})(1-\frac{p}{p+\Sigma\Sigma r_{ij}})$ where p denotes the number of items and r_{ij} denotes the polychoric correlation between items i and j. Polychoric correlation assumes that the two items follow bivariate normal distribution which needs to be tested by goodness of fit tests like the likelihood ratio, χ^2 test, G^2 test, etc. making further assumptions that under the null hypothesis data come from a Multivariate distribution. The degree of deviations from bivariate normality may result in biased estimate of polychoric correlations. Babakus, Ferguson and Jöreskog (1987) found that Polychoric correlations performed worst on all goodness–of–fit criteria. However, distribution of underlying variables can be highly skewed and this may introduce bias in the result of χ^2 test to assess goodness of fit of structural equation models (Muthen, 1993). Moreover, the polychoric correlation matrix may be non-positive definite. For small samples, polychoric correlation offers a rather unstable estimate. Even for large samples, the estimates are noisy if there are few empty cells. In case of items with smaller number of response categories, polychoric correlation between latent continuous variables tends to be attenuated. However, reliability using polychoric correlation is not a non-parametric approach because of the assumption of bivariate normality of the underlying variables.

Lewis (2007) referred the ordinal reliability as nonparametric reliability coefficients in a nonlinear classical test theory sense even though such reliabilities assume that the underlying variable is continuous. Zumbo, Gadermann and Zeisser (2007) suggested a measure of reliability viz. Coefficient theta proposed by Armor (1974) that is based on principal components analysis. If the single factor solution is reasonable for the items, then $\theta = (\frac{p}{p-1})(1-\frac{1}{\lambda_1})$where $\lambda_1$is the largest eigen value obtained from the principal component analysis (PCA) of the correlation matrix for the items. However, estimation of λ_1 based on the sample covariance matrix is extremely sensitive to outlying observations. PCA relies on linear assumptions. But the data may not always be linearly correlated.

This state of affairs motivates a need to find methods of obtaining reliability of Likert items and Likert scale from a single administration of the questionnaire using only the permissible operations for a Likert scale i.e. considering the cell frequencies or empirical probabilities of Item – Response categories without making any assumptions of continuous nature or linearity or normality for the observed variables or the underlying variable being measured.

2. Objectives

To find non-parametric methods of obtaining reliability of Likert items and Likert scale from a single administration of the questionnaire using only the permissible operations for a Likert scale i.e. considering the cell frequencies or empirical probabilities of Item – Response categories without making any assumptions of continuous nature or linearity or normality for the observed variables or the underlying variable being measured.

3. Methodology

Suppose there are *n* respondents who answered each of the *m* items of a Likert questionnaire where each item had *k* numbers of response categories. Consider the basic data matrix X=$\left(\left(X_{ij}\right)\right)$ where X_{ij} represents score of the *i*-th individual for the *j*-th item, *i*= *1,2,….,n* and *j=1,2,….m*. Value of X_{ij} ranges between *1* to k and $\sum_{i=1}^{n} X_{ij}$ = Sum of scores of all individuals for the *j*-th item (Item Score for the *j*-th item)

It is possible to have another frequency matrix ***F***= ((f_{ij})) of order *m* X *k* showing frequency of *i*-th item and *j*-th response category. A row total will indicate frequency of that item and will be equal to the sample size (*n*). Similarly, a column total will indicate total number of times that response category was chosen by all the respondents. Denote the column total of *j*-th response category by f_{0j} for *j = 1, 2, 3…k*. Here, $\sum_{j=1}^{m} X_{ij}$ = Sum of scores of all the items for *i*-th individual i.e. total score of the *i*-th individual (Individual score) and $\sum_{j=1}^{k} f_{0j}$ = Grand total = (Sample size) (number of items) =mn. Clearly, $\sum\sum X_{ij}$ = Sum of scores of all the individuals on all the items i.e. total test score.

After administration of the questionnaire to a large number of respondents, one can calculate *k*-dimensional vector of empirical probabilities for the *i*-th item with *k*-response categories as$\boldsymbol{P_i} = (p_{i1}, p_{i2}, \dots \dots \dots \dots, p_{ik})^T$. Clearly$\sum_{j=1}^{k} p_{ij} = 1$. Similarly, for the entire questionnaire, vector showing empirical probabilities will be $\boldsymbol{T} = (\frac{f_{01}}{mn}, \frac{f_{02}}{mn}, \frac{f_{03}}{mn}, \dots \dots., \frac{f_{0k}}{mn})^T$ and $\sum_{j=1}^{k} \frac{f_{oj}}{mn} = 1$.

3.1 Proposed methods

Two methods based on angular associations i.e. cosines of angle between two vectors are proposed below:

3.1.1 Cosine similarity Method

Popular measure of similarity between two *n*- dimensional vectors ***X*** and ***Y*** are the Jaccard measure $J(X.Y)$,Dice measure$D(X.Y)$ and Cosine similarity measure $C(X.Y)$ defined as : $J(X.Y) = \frac{X.Y}{\|\boldsymbol{X}\|^2 + \|\boldsymbol{Y}\|^2 - \boldsymbol{X.Y}}$; $D(X.Y) = \frac{2\boldsymbol{X.Y}}{\|\boldsymbol{X}\|^2 + \|\boldsymbol{Y}\|^2}$ and $C(X.Y) =$

$\frac{\boldsymbol{X.Y}}{\|\boldsymbol{X}\|\|\boldsymbol{Y}\|}$. All the three measures are similar in the sense they consider dot product of two vectors, take values in the interval [0,1] for acute angle between the two vectors and $J(X.Y) = D(X.Y) = C(X.Y) = 1$ if and only if $\boldsymbol{X} = \boldsymbol{Y}$

Thada and Jaglan (2013) found that for a general dataset, $C(X.Y) > D(X.Y) > J(X.Y)$ and best fit values were obtained using $C(X.Y)$. Use of Cosine similarity is common in areas like information retrieval and text mining, involving higher dimensional spaces (Singhal, 2001). Accordingly, association between *i*-th and *j*-th item can be taken as $Cos\theta_{ij}$ where θ_{ij} is the angle between the vectors $\boldsymbol{P_i}$ and $\boldsymbol{P_j}$ and to be computed as

$$Cos\theta_{ij} = \frac{P_i^T P_j}{\|P_i\|\|P_j\|} \quad \text{........} \quad (1.1)$$

Similarly, Item-test correlation between the *i*-th item and total score can be obtained by $Cos\theta_{iT}$ where θ_{iT} is the angle between the vectors $\boldsymbol{P_i}$ and $\boldsymbol{T}$

$$Cos\theta_{iT} = \frac{P_i^T.T}{\|P_i\|\|T\|} \quad \text{.......} \quad (1.\ 2)$$

Note that $Cos\theta_{ij}$ as defined in (1.1) satisfy the following:

- If $P_i = P_j$ where $i \neq j$ then $Cos\theta_{ij} = 1$ and vise versa.
- $Cos\theta_{ij} = 0$ if and only if the vectors $\boldsymbol{P_i}$ and $\boldsymbol{P_j}$ are orthogonal
- Symmetric i.e. $Cos\theta_{ij} = Cos\theta_{ji}$
- Satisfy non-negativity condition i.e. $Cos\theta_{ij} \geq 0$.
- Does not satisfy triangle inequality i.e. it does not satisfy $Cos\theta_{XY} + Cos\theta_{YZ} \geq Cos\theta_{XZ}$ where $X \neq Y \neq Z$. In other words, $Cos\theta_{ij}$ is not a metric.

Correlation or association between a pair of Item in terms of $Cos\theta_{ij}$ is always non-negative. Thus, the method helps to avoid negative correlations between a pair of items. Item reliability in terms of correlation between an item and the test in terms of $Cos\theta_{iT}$ will always be positive. Test reliability should not be computed as average of $Cos\theta_{ij}'s$ or $Cos\theta_{it}'s$ since $Cos\theta_{ij}$ does not obey triangle inequality. The symmetric matrix showing $Cos\theta_{ij}'s$ may be used to find value of test reliability and also to undertake factor analysis. However, $Cos\theta_{iT}$ will indicate reliability of the *i*-th item.

Following Gaddernan, Guhn and Zumbo (2012), reliability of a test with m items can be found by replacing the polychoric correlation between items i and j by $Cos\theta_{ij}$ in the following equation

$$r_{tt} = \frac{m}{m-1}\left(1 - \frac{m}{m+\sum\sum Cos\theta_{ij}}\right) \quad \text{......................} \quad (1.3)$$

Clearly, equation (1.3) requires computation of inter-item correlation matrix in terms of $Cos\theta_{ij}$. It may be noted that test reliability as per equation (1.3) is not a function of item reliabilities.

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3.1.2 Bhattacharyya's measure

To make $\boldsymbol{P_i}$ and $\boldsymbol{P_j}$ as unit vector, one may choose $\boldsymbol{\pi_i}$ and $\boldsymbol{\pi_j}$ where $\boldsymbol{\pi_i} = \sqrt{\frac{\boldsymbol{P_i}}{\|P_i\|}}$ and $\boldsymbol{\pi_j} = \sqrt{\frac{\boldsymbol{P_j}}{\|P_j\|}}$ so that $\|\pi_i\|^2 = \|\pi_j\|^2 = 1$. Association between the *i*-th item and *j*-th item i.e. association between vector $\boldsymbol{P_i} = (p_{i1}, p_{i2}, p_{i3}, p_{i4}, \ldots\ldots p_{ik})^T$ and vector $\boldsymbol{P_j} = (p_{j1}, p_{j2}, p_{j3}, p_{j4}, \ldots\ldots p_{jk})^T$ can be found by Bhattacharyya's measure (Bhattacharyya, 1943) as cosine of the angle $\emptyset_{ij}$ where $\emptyset_{ij}$ is the angle between the two vectors $\boldsymbol{\pi_i}$ and $\boldsymbol{\pi_j}$ since $\|\boldsymbol{\pi_i}\|^2 = \|\boldsymbol{\pi_j}\|^2 = 1$. The Bhattacharyya's measure is in fact a measure of similarity between $\boldsymbol{P_i}$ and $\boldsymbol{P_j}$.

Thus, $\rho\left(\boldsymbol{\pi_i}.\boldsymbol{\pi_j}\right) = Cos\,\emptyset_{ij} = \sum_{s=1}^{k}\sqrt{\pi_{is}\pi_{js}}$ ….. ….. (1.4)

$$\text{where } \pi_{is} = \sqrt{\frac{p_{is}}{\|P_i\|}} \quad \forall\, i = 1,2,\ldots,m \text{ and } s = 1,2,\ldots\ldots k$$

Item reliability in terms of Item-test correlation using Bhattacharyya's measure can be defined as

$$Cos\emptyset_{iT} = \rho(\boldsymbol{\pi_i}.\sqrt{\boldsymbol{T}}) = \sum_{j=1}^{k}\sqrt{\frac{f_{Ij}}{mn}p_{ij}} \quad \ldots \quad \ldots. \qquad (1.5)$$

It can be proved easily that

i) The measure is defined even if a p_{ij} is equal to zero i.e. if all respondents do not choose a particular response category of an item.

ii) $\rho\left(\boldsymbol{\pi_i}.\boldsymbol{\pi_J}\right) = 1$ if the vectors $\boldsymbol{\pi_i}$ and $\boldsymbol{\pi_J}$ are identical

= 0 if $\boldsymbol{\pi_i}$ and $\boldsymbol{\pi_J}$ are orthogonal

iii) $0 \leq \rho\left(\boldsymbol{\pi_i}.\boldsymbol{\pi_j}\right) \leq 1$ using Jensen's inequality

iv) $\rho\left(\boldsymbol{\pi_i}.\boldsymbol{\pi_j}\right) > Cos\theta_{ij}$. i.e. inter-item correlations as per Bhattacharyya's measure are greater than the same obtained from Angular association method.

v) Does not satisfy triangle inequality.

While dealing with vectors of unit length, Rao (1973) has shown that mean and dispersion of the angles $\emptyset_1, \emptyset_2, \emptyset_3, \ldots\ldots\ldots \emptyset_k$ can be found as follows:

Mean or most preferred direction is estimated by $\overline{\emptyset} = Cot^{-1}\frac{\sum\cos\emptyset_i}{\sum\sin\emptyset_i}$ and the dispersion by $\sqrt{1-r^2}$ where $r^2 = (\frac{\sum Cos\emptyset i}{k})^2 + (\frac{\sum Sin\emptyset_i}{k})^2$. Reliability of the Likert scale can be defined as $Cos(\overline{\emptyset})$ = $Cos\ (Cot^{-1}\frac{\sum\cos\emptyset_i}{\sum\sin\emptyset_i})$…. …. (1.6)

The above will help to find reliability of the Likert scale as a function of item reliabilities where range of reliability can be found from $(Cos\overline{\emptyset} \pm C\sqrt{1-r^2}$) where C is a suitably chosen constant.

4. Empirical verification

A questionnaire consisting of five Likert items each with five response alternatives was administered to 100 respondents where "Strongly agree" was assigned 5 and "Strongly disagree" was assigned 1. Here, m = 5, *k* = 5 and *n* = 100

Table - 1
Item – Response Categories frequency matrix and Probabilities

Items	Frequency/ Probability	RC-1	RC- 2	RC- 3	RC- 4	RC- 5	Total
1	Frequency	19	32	35	11	3	100
	Probability(P_{1j})	*0.19*	*0.32*	*0.35*	*0.11*	*0.03*	*1.00*
2	Frequency	7	33	34	19	7	100
	Probability(P_{2j})	*0.07*	*0.33*	*0.34*	*0.19*	*0.07*	*1.00*
3	Frequency	34	11	5	14	36	100
	Probability(P_{3j})	*0.34*	*0.11*	*0.05*	*0.14*	*0.36*	*1.00*
4	Frequency	10	14	38	30	8	100
	Probability(P_{4j})	*0.10*	*0.14*	*0.38*	*0.30*	*0.08*	*1.00*
5	Frequency	4	31	37	20	8	100
	Probability(P_{5j})	*0.04*	*0.31*	*0.37*	*0.20*	*0.08*	*1.00*
Total	Frequency(f_{0i})	74	121	149	94	62	500
	Probability($\frac{f_{0i}}{mn}$)= (P_{iT})	*0.148*	*0.242*	*0.298*	*0.188*	*0.124*	*1.00*

Legend: RC- j denotes j-th Response Category $\forall\, j = 1,2,\ldots,5$

4.1 Descriptive statistics for the usual summative method for the items and test obtained from the usual summative methods are as follows:

Table – 2
Mean, variance, Skewness and Kurtosis of items and test

	Mean	Variance	Skewness	Kurtosis
Item - 1	2.47	1.0395	0.2870	-0.3495
Item - 2	2.86	1.0711	0.2865	-0.4669
Item - 3	2.94	1.2489	-0.2343	-0.6592
Item - 4	3.12	1.1572	-0.3429	-0.3524
Item - 5	2.97	0.9991	0.3087	-0.4476
Test	14.36	6.1923	0.0436	0.2134

Observations:

- Item – 3 had maximum variance.
- Values of skewness and kurtosis were different from zero for each item which implies that item score are not normally distributed.

4.2 Item correlation matrix and item-test correlations as obtained from the three methods are given in Table – 3

Table – 3
Item correlation matrix

	Item-1	Item-2	Item-3	Item-4	Item-5	Test
A. Usual Summative Method						
Item-1	1.00	(-)0.0040	0.0782	0.1230	0.0834	0.5298
Item-2		1.00	0.1149	(-) 0.1662	0.0838	0.4277
Item-3			1.00	(-) 0.0528	0.1431	0.5636
Item-4				1.00	(-) 0.0906	0.3535
Item-5					1.00	0.4958
B. **Cosine Similarity Method**						

Item-1	1.00	0.9585	0.5186	0.8531	0.9395	0.9542
Item-2		1.00	0.4690	0.9061	0.9958	0.9674
Item-3			1.00	0.5064	0.4390	0.6567
Item-4				1.00	0.9229	0.9358
Item-5					1.00	0.9601
C. **Bhattacharyya's measure**						
Item-1	1.00	0.9754	0.8021	0.9448	0.9593	0.9737
Item-2		1.00	0.7970	0.9716	0.9972	0.9848
Item-3			1.00	0.8210	0.7748	0.8831
Item-4				1.00	0.9715	0.9793
Item-5					1.00	0.9764

The above table revels:

For usual summative method**, i**tem correlations were low and few were found to be negative and item-test correlations ranged between 0.35 to 0.56. However, for Cosine Similarity method and Bhattacharyya's measure, all item correlations in terms of $Cos\theta_{ij}$ and $Cos\ \emptyset_{ij}$ respectively were positive and value of item-correlation between *i*-th and *j*-th item was more than the same for usual summative method $\forall i, j = 1,2, \dots \dots ..,5$. Same was true for item-test correlations also.

4.3 Reliability of items and the test for each of the above method was computed and are shown below:

Table – 4
Item reliability and test reliability for different approaches

Item No.	Summative method	Cosine similarity method	Bhattacharyya's measure
	Item reliability (r_{iT}) and test reliability (Cronbach's alpha)	Item reliability ($Cos\theta_{iT}$) and test reliability as per (1.3)	Item reliability ($Cos\emptyset_{iT}$) and test reliability as per (1.6)
1	0.5298	0.9542	0.9737

2	0.4277	0.9674	0.9848
3	0.5636	0.6567	0.8831
4	0.3535	0.9358	0.9793
5	0.4958	0.9601	0.9764
Test	0.1366	0.9378	0.9899

Reliability of items and the test increased by each of the two proposed methods in comparison to the usual summative method.

Sum of inter-item correlations excluding the diagonal elements, $\sum\sum Cos\theta_{ij}$ for i$\neq$ j is 2(7.508924) = 15.017848. Reliability of the test as per Cosine similarity method was $r_{tt} = \frac{m}{m-1}\left(1 - \frac{m}{m+\sum\sum Cos\theta_{ij}}\right)$= 0.9378 which is much greater than Cronbach's alpha for the test.

As per Bhattacharyya's measure , $\sum_{i=1}^{5} Cos\emptyset_{iT}$= 9.01455. Corresponding value of $\sum_{i=1}^{5} Sin\emptyset_{iT}$= 1.28869. Using (1.6), test reliability is $Cos(\overline{\emptyset})$ = $Cos\ (Cot^{-1}\frac{\sum\cos\emptyset_i}{\sum\sin\emptyset_i})$

= $Cos\ (Cot^{-1}6.9951) = Cos\ (8.135721\ degrees) = 0.9899$

Reliability of the test as per Bhattacharyya's measure was found to be highest among the three methods discussed here.

4.4 Effect of deletion of items on reliability for the three methods were computed and details are shown below.

Table – 5
Effect of deletion of Item on Test Reliability

Description	Cronbach's alpha	Cosine similarity method Test reliability as per (1.3)	Bhattacharyya's measure Test reliability as per (1.6)
Test with 5 items	0.1366	0.9378	0.9899
If Item – 1 is deleted	0.0197	0.9059	0.9702
If Item – 2 is deleted	0.1623	0.4876	0.9617
If Item – 3 is deleted	0.0123	0.7764	0.9897
If Item – 4 is deleted	0.2684	0.9114	0.9686
If Item – 5 is deleted	0.0588	0.4293	0.9645

It may be observed that Cronbach's alpha increased on deletion of Item – 2 and also Item – 4 and thus, test reliability in terms of α may not be robust.

Value of test reliability as per Cosine similarity method and Bhattacharyya's measure exceeded alpha significantly. Maximum value of test reliability was obtained while using Bhattacharyya's measure.

Deletion of an item resulted in decrease of test reliability as per Cosine similarity method and Bhattacharyya's measure also. However, fluctuations of reliabilities upon deletion of an item did not show any pattern for each of the two proposed method. Thus, the Bhattacharyya's measure showed more robustness of reliability.

4.4 Summary of comparison of the three methods is given below:

Table - 6
Summary of comparison of the three methods

Description	Summative method	Cosine similarity method	Bhattacharyya's measure
Assumptions	Data are continuous, uncorrelated errors and normally distributed	No assumption of continuous nature or linearity or normality for the observed variables or the underlying variable being measured	No assumption of continuous nature or linearity or normality for the observed variables or the underlying variable being measured
Avoids		Test of uni-dimensionality or bivariate normality associated with the polychoric correlations.	Test of uni-dimensionality or bivariate normality associated with the polychoric correlations
Item correlations	-Found to be positive and also negative	-Always positive. -More homogeneous	-Always positive. -Highest among three methods -More homogeneous
Item-test correlation	Maximum 0.5636 Minimum 0.3535	Maximum 0.9674 Minimum 0.6567	Maximum 0.9848. Minimum 0.8831
Test reliability considers	Item variances, test variance and number of items	Cell frequencies or empirical probabilities of Item – Response categories, number of items, Inter-item	Cell frequencies or empirical probabilities of Item – Response categories, number of items, Item reliabilities

		correlations	
Numerical value of reliability	Low $\alpha = 0.1366$	Higher $r_{tt} = 0.9378$	Highest $r_{tt} = 0.9899$
Reliability on deletion of an item	Increased on deletion of item 2 and also on deletion of item 4.	Did not increase on deletion of any one item	-Did not increase on deletion of any one item -Most robust
Test reliability as a function of item reliabilities	Not possible	Not possible	Possible

5. Findings and Conclusions

Reliability of a Likert scale and Likert item were found by Cosine similarity method and Bhattacharyya's measure, using only the frequencies of Item – Response categories without involving assumptions of continuous nature or linearity or normality for the observed variables or the underlying variable being measured. Thus, such reliabilities are in fact Non-parametric and suitable alternatives to coefficient alpha to compute reliability of Likert response data. The proposed methods also avoid test of unidimensionality or assumption of normality for Cronbach's alpha or bivariate normality associated with the polychoric correlations. The problem of outlying observations and reliance on linear assumptions associated with PCA for finding reliability theta are also avoided in each of the proposed method. Thus, the proposed methods are considered as improvement over the existing ones. The methods help the researchers to find better estimates of Likert reliabilities in non-parametric ways.

Reliability of the test for Cosine similarity method replaced polychoric correlation between items i and j by $Cos\theta_{ij}$ which can be computed irrespective of nature of distributions of the observed or underlying variables or factor structure. Value of test reliability by this Cosine similarity method and by Bhattacharyya's measure was found to be 0.94 and 0.99 respectively against Cronbach's α of 0.14 only. Range of Item reliability was found to be highly desirable both in Cosine similarity method and Bhattacharyya's measure. Significant values of the elements of the Inter-Item correlation matrix tend to indicate inter-item consistency leading to possible uni-dimensionality which may be confirmed through factor analysis.

Test reliability by Bhattacharyya's measure has a special property as it can be expressed as a function of Item reliabilities. The approach also helps to find range of reliability of the entire Likert scale.

Ranks of Items in terms of Item reliabilities were different for different methods. Test reliability did not increase on deletion of any item in the Cosine similarity method and for Bhattacharyya's measure. Empirically, the Bhattacharyya's measure showed maximum robustness of reliability. Thus, reliability as per Bhattacharyya's measure appears to be preferred among the three methods discussed. Use of Non-parametric reliability by Bhattacharyya's measure is recommended for Likert-type data for clear theoretical advantages.

Further studies may be undertaken to find item reliability and reliability of Likert scale and to facilitate comparison of the two proposed methods with other existing methods.

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